TESTING OF HYPOTHESIS

Hypothesis testing is a statistical method that is used in making statistical decisions using experimental data. Hypothesis testing is basically an assumption that we make about the population parameter.

Types of hypothesis

There are two types of hypothesis namely

Null hypothesis

Null hypothesis is a statistical hypothesis that assumes that the observation is due to a chance factor. Null hypothesis can be defined as the hypothesis of no difference. It is denoted by $H_{0.}$

Alternative hypothesis

Contrary to the null hypothesis, the alternative hypothesis shows that observations are the result of a real effect.

Level of significance

Refers to the degree of significance in which we accept or reject the null hypothesis. 100% accuracy is not for accepting or rejecting a hypothesis so we therefore select a level of significance that is usually 5%. Varying degree of certainty or significance to which we have referred (5%,1%) are called as level of significance.

One tailed test

A one tailed test is a statistical test in which the critical area of a distribution is one sided so that it is either greater than or less than a certain value, but not both. One tailed test is also known as a directional hypothesis or directional test.

Two tailed test

A two tailed test is a statistical test in which the critical area of a distribution is two sided and tests whether a sample is greater than or less than a certain range of values.

Degree of Freedom

Degree of freedom is the number of independent values in the sampling distribution.

Test of Significance for Difference of Means

Different methods are used for comparing the means of two groups for independent and dependent samples.

TYPES OF t TESTS

The t tests are of three types : t test between a sample and a population mean, t test for independent groups, and t test for dependent groups.

Assumptions for Using Student's t Test

1. The population from which the sample is drawn is normal.
2. The sample drawn is random
3. The population standard deviation σ is unknown.

INDEPENDENT t TEST

Suppose we want to test the significance of the difference between the means of the two independent samples. Then under the null hypothesis $H_0\ \mu_x = \mu_y$ and under the assumption that the population variance are equal

i.e. $\sigma_x^2 = \sigma_y^2 = \sigma^2$ (say)

the statistic

$$t = \frac{\bar{x} - \bar{y}}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

follows student's t-distribution with (n_1+n_2-2) d.f.

Where $\mu_x,\ \sigma_x{}^2$ and $\mu_y,\ \sigma_y{}^2$ are the mean and variances of the two groups and $\bar{x}$ and $\bar{y}$ are the means of the two samples and

$$s^2 = \frac{1}{n_1 + n_2 - 2}\left[\sum_i (x_i - \bar{x})^2 + \sum_j (y_j - \bar{y})^2\right]$$

Assumptions of t- test for Difference of Means

While using t-test for testing difference of means we make the following three fundamental assumptions:

a. Parent populations, from which the samples have been drawn are normally distributed.

b. The population variance of the groups are equal and unknown, i.e. $\sigma_A^2 = \sigma_B^2 = \sigma^2$

Example The vertical jump performance of 10 randomly chosen college volleyballers in cms are 64,58,62,58,60,59,50,58,65,59 and those of 10 randomly chosen college basketballers are 60,48,60,68,58,59,55,65,56,50. Test if the two groups differ significantly in reaction to the performance on vertical jump.

Hypothesis:

H_0 $\mu_x = \mu_y$

H_1 $\mu_x \neq \mu_y$

LEVEL OF SIGNIFICANCE

.05

TEST STATISTIC

$$t = \frac{\overline{x} - \overline{y}}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$s^2 = \frac{1}{n_1 + n_2 - 2}\left[\sum_i (x_i - \overline{x})^2 + \sum_j (y_j - \overline{y})^2\right]$$

$$\overline{X} = 59.3$$
$$\overline{Y} = 57.9$$
$$s = 5.21$$
$$t = 0.60$$

For the two tailed test tabulated t =2.10

Inference

Since calculated t is less than tabulated t at 5% level of significance, we may accept H_0 Thus it may be concluded that the vertical jump performance of both volley ball and basketball groups are same.

DEPENDENT t TEST

Let us consider a situation where the samples have equal sizes and the two samples are not independent but the sample observations are paired together. The problem is to test if the sample means differ significantly or not?

Example A 4 weeks aerobic workout was given to ten females. Their haemoglobin contents were measured before and after the 4 weeks programme and are shown below. Can it be concluded at 5% level of significance that the aerobic workout in general increases the haemoglobin contents?

Pre test	Post test	D	D^2
15.2	15.1	-.1	.01
11.8	12.4	.6	.36
12.8	15.6	2.8	7.84
11.7	15.4	3.7	13.69
13.4	15.2	1.8	3.24
13.8	14.4	.6	.36
13.9	16.2	2.3	5.29
12.2	12.4	.2	.04
12.6	15.8	3.2	10.24
12.6	12.4	-.2	.04
		14.9	41.11

Solution

Since subjects are same during pre and post testing therefore groups are related. In this case paired t-test shall be used to test the hypothesis

Hypothesis:

H_0 $\mu_x = \mu_y$

H_1 $\mu_x < \mu_y$

LEVEL OF SIGNIFICANCE

.05

TEST STATISTIC

$$t = \frac{\overline{D}}{s/\sqrt{n}}$$

$$\overline{D} = \frac{\sum D}{n} = \frac{14.9}{10}$$

$$s = \sqrt{\frac{1}{10-1}(41.11 - 10 \times 1.49^2)} = 1.45$$

$$t = \frac{\overline{D}}{s/\sqrt{n}} = 3.25$$

For one tailed test t=1.83

Inference

Since calculated t is greater than tabulated t , H_0may be rejected at 5% level of significance. Thus it may be concluded that aerobic workout increases the haemoglobin contents.

USES OF t-DISTRIBUTION

The t-distribution has a large number of application in research, some of which are listed below :-

a) To test whether the sample mean x is significantly different from the hypothetical value of μ of the population mean.

b) To test the significance of the difference between two sample means.

c) To test the significance of an observed sample correlation coefficient.

d) To test the significance of an observed partial and multiple correlation coefficient.

Z test

A z-test is a statistical test used to determine whether two population means are different when the variances are known and the sample size is large. The test statistic is assumed to have a normal distribution, and nuisance parameters such as standard deviation should be known in order for an accurate z-test to be performed.

Two-Sample z-test for Comparing Two Means

Requirements: Two normally distributed but independent populations, σ is known

Hypothesis test

Formula:

$$z = \frac{\bar{x}_1 - \bar{x}_2 - \Delta}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

where $\bar{x}_1$ and $\bar{x}_2$ are the means of the two samples, Δ is the hypothesized difference between the population means (0 if testing for equal means), σ_1 and σ_2 are the standard deviations of the two populations, and n_1 and n_2 are the sizes of the two samples.

The amount of a certain trace element in blood is known to vary with a standard deviation of 14.1 ppm (parts per million) for male blood donors and 9.5 ppm for female donors. Random samples of 75 male and 50 female donors yield concentration means of 28 and 33 ppm, respectively. What is the likelihood that the population means of concentrations of the element are the same for men and women?

Null hypothesis: H_0: $\mu_1 = \mu_2$

or H_0: $\mu_1 - \mu_2 = 0$

alternative hypothesis: H_a : $\mu_1 \neq \mu_2$

or: H_a : $\mu_1 - \mu_2 \neq 0$

$$z = \frac{28 - 33 - 0}{\sqrt{\frac{14.1^2}{75} + \frac{9.5^2}{50}}} = \frac{-5}{\sqrt{2.65 + 1.81}} = -2.37$$

The computed *z*-value is negative because the (larger) mean for females was subtracted from the (smaller) mean for males. But because the hypothesized difference between the populations is 0, the order of the samples in this computation is arbitrary— $\bar{x}_1$ could just as well have been the female sample mean and $\bar{x}_2$ the male sample mean, in which case *z* would be 2.37 instead of –2.37. An extreme *z*-score in either tail of the distribution (plus or minus) will lead to rejection of the null hypothesis of no difference.

The area of the standard normal curve corresponding to a *z*-score of –2.37 is 0.0089. Because this test is two-tailed, that figure is doubled to yield a probability of 0.0178 that the population means are the same. If the test had been conducted at a pre-specified significance level of $\alpha < 0.05$, the null hypothesis of equal means could be rejected. If the specified significance level had been the more conservative (more stringent) $\alpha < 0.01$, however, the null hypothesis could not be rejected.

In practice, the two-sample *z*-test is not used often, because the two population standard deviations σ_1 and σ_2 are usually unknown. Instead, sample standard deviations and the *t*-distribution are used.